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Microwaves

A summary on dB (decibel)

In general, an attenuation or gain is defined as 10 times the logarithm in base 10 of the ratio between an output power (P_2) to an input power (P_1) between two ports of an N-port device.

$$\text{Attenuation: } A = 10\log_{10}\frac{P_1}{P_2} = 10\log_{10} a \qquad P_1 \geq P_2$$

$$\text{Gain: } G = -A = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10} g \qquad P_1 \geq P_2$$

Power ratios expressed in decibels (dB) are used in telecommunication in order to facilitate planning work. Let us consider for instance Friis' formula :

$$\frac{P_r}{P_e} = g_e g_r \left(\frac{\lambda}{4\pi R}\right)^2$$

Which becomes in dB

$$10\log_{10}\frac{P_r}{P_e} = G_e + G_r + 10\log_{10}\left(\frac{\lambda}{4\pi R}\right)^2 = G_e + G_r + 20\log_{10}\left(\frac{\lambda}{4\pi R}\right)$$

$$G = 10\log_{10} g$$

We see that the introduction of the logarithms allows us to replace multiplications by additions.

In some cases, the power ratios are expressed as a function of voltage, current or field ratios. These expressions being proportional to the square root of the power, we have:

$$10\log_{10}\frac{P_1}{P_2} = 20\log_{10}\frac{I_1}{I_2} = 20\log_{10}\frac{U_1}{U_2} = 20\log_{10}\frac{E_1}{E_2} = 20\log_{10}\frac{H_1}{H_2}$$

Note that the dB is dimensionless, at it expresses a ratio of power. However, in order to fully use the simplifications brought by the logarithmitic way to write the expressions, we need often to express powers or fields in a logarithmic way also. To do this, we normalize the power by dividing it by a known power (typically a Watt or a milliwatt), before taking the logarithm of this ratio and multiply it by 10. We then add a letter after the dB which indicates which normalization was chosen :

$$1\left[W\right] \text{becomes: } 10\log_{10}\frac{1\left[W\right]}{1\left[W\right]}\,dBW = 10\log_{10}\frac{1000\left[mW\right]}{1\left[mW\right]}\,dBm$$

The gain of an antenna is in general obtained by comparing the power density emitted by the antenna to the power density emitted by an isotropic antenna fed by the same input power. In this case the ratio of power densities should be expressed in dB, as no normalization was necessary. However we add the letter i after dB in this case, in order to stipulate that the reference antenna is isotropic. Indeed, in some cases a dipole antenna is used as reference instead of an isotropic antenna. In this case, the letter d is added to dB:

dBi for a gain obtained with an isotropic reference
dBd for a gain obtained with a dipole reference

x dBi =(x + 2.15) dBd